

MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2024 (full-time)

Assignment 3

Due Date: May 21 (in class)

Instruction

- (a) You can answer in English or Chinese or both.
 - (b) Show **enough** intermediate steps.
 - (c) Write your answers **independently**.
 - (d) If you copy the solutions from somewhere, you must **indicate the source**.
-

Question 1 (15 + 10 = 25 points)

Fibonacci Generator is a simple Multiple Recursive Generator (MRG). It extends the Linear Congruential Generator (LCG) in the following way:

$$x_i = (x_{i-1} + x_{i-2}) \bmod m.$$

But Fibonacci Generator has a serious flaw in statistics.

- (1) Prove that Fibonacci Generator can never successively generate three pseudo-random numbers u_i, u_{i+1}, u_{i+2} satisfying $u_i < u_{i+2} < u_{i+1}$.
- (2) Prove that for a perfect RNG, the probability of the event stated in (1) should be $1/6$.

Question 2 (20 points)

Rigorously prove the Box-Muller method (see Lec 4 page 35/38).

Question 3 (20 points)

Suppose $\{X_1, \dots, X_n\}$ is a random sample from $\mathcal{N}(\mu, \sigma^2)$. Find the estimators of μ and σ^2 using MLE. (*Write down the rigorous derivation steps.*)

Question 4 (5 + 10 = 15 points)

For the illustrative example on lecture note Lec 5, the first considered exponential distribution is rejected. We then consider the Weibull family. The density function of Weibull(α, β) in shape & scale parametrization is $f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha}$, $x > 0, \alpha > 0, \beta > 0$. Suppose the parameters are estimated from the data via MLE: $\hat{\alpha} = 0.525$, $\hat{\beta} = 6.227$.

- (1) Make the Q-Q plot. (*Note:* Show the necessary calculation. Use Excel or other software/language to draw the final plot.)
- (2) Use K-S test to see if we would like to reject Weibull(0.525, 6.227) at level of significance $\alpha = 0.1, 0.05, 0.01$. (*Note:* You can use Excel or other software/language to compute the value of test statistic D ; but implement the formula of D by yourself. The $(1 - \alpha)$ -quantile of D is $d_{n,1-\alpha} = c/\sqrt{n}$, and the value of c is given in the following table.)

n	$1 - \alpha$			
	0.900	0.950	0.975	0.990
10	0.679	0.730	0.774	0.823
20	0.698	0.755	0.800	0.854
50	0.708	0.770	0.817	0.873
∞	0.715	0.780	0.827	0.886

Question 5 (20 points)

Suppose we are modeling one simulation input. We have collected 2000 independent observations (see the `data.xlsx` file). For this input, we only know it is a continuous variable. Finish the steps 2–4 for input modeling (Lec 5, page 5/57). In step 4, just use one graphical method.